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THE USE OF UTILITY IN MULTIATTRIBUTE UTILITY ANALYSIS: (U)

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THE USE OF UTILITY IN MULTIATTRIBUTE UTILITY ANALYSIS,

Alain A. Lewis

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THE USE OF UTILITY IN MULTIATTRIBUTE UTILITY ANALYSIS

Alain A. Lewis

This paper was written at Harvard University for a Project on Efficiency in Decision Making.

# ABSTRACT

A frequent source of confusion in the application of the techniques of Multiattribute Utility Analysis is the failure to observe the distinction between value functions and utility functions. A concise exposition of this distinction is provided along with the relationships to alternative measures of preference rankings.

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There seems to be a frequently recurring confusion in the use of Multi-Attribute Utility Analysis as set forth in the recent treatise of Ralph L. Keeney and Howard Raiffa, Decisions with Multiple Objectives, Wiley & Sons, 1976 between value functions and utility functions. Very often, arguments arise falsely over issues of methodology which vanish when the appropriate distinctions between the two varieties of functions are made and observed.

We can consider both classes of functions as a form of representation of a decision maker's preference over an abstract space of alternatives, (possibly infinite)  $X$ . More precisely, if  $X$  can be ordered by a binary relation,  $R$ , where for  $(x', x) \in X \times X$ ,  $x'Rx$  is to mean the decision maker "prefers"  $x'$  to  $x$ , a real-valued (and usually continuous) function,  $f: X \rightarrow \mathbb{R}_+$  [or  $f: X \rightarrow (0, 1)$ ] is said to "represent" the preference  $R$  on  $X$ , if it preserves the order on  $X$  induced by  $R$ , i.e.  $f(x') > f(x)$  if and only if  $x'Rx$ . (This is very often "weakened" to "faithful" representations,  $f(x') > f(x)$  if  $x'Rx$ ).

Allow next  $U$  to be the class of utility functions and  $V$  to be the class of value functions. Then the distinction (or perhaps relationship) between  $U$  and  $V$  is that  $U \subset V$  representations of  $R$ . Therefore, if  $f: X \rightarrow (0, 1)$  and  $f \in U$  then  $f \in V$ , but not conversely. Mathematically, this is expressed as saying that value functions are ordinal representations whereas utility functions are cardinal representations, and that the of class ordinal representations strictly includes the class of cardinal representations. Very often, it is also said that a utility function measures preference, whereas a value function merely discerns preference. Clearly, if a preference can be measured, it can therefore be discerned, but not all discernments are measurable.

As no uniformly acceptable terminology exists for this distinction, very often these different concepts are referred to by the same name. Keeney and Raiffa point this out in a footnote:

"Unfortunately, there is no standardized terminology for what we have chosen to call value functions and utility functions. In the literature, our value functions are sometimes referred to as worth functions, ordinal utility functions, preference functions, Marshallian utility functions, and even utility functions. Similarly, our utility functions are referred to as preference functions, cardinal utility functions, Von Neumann utility functions, probabilistic utility functions, and utility functions. Although clearly we cannot be consistent with all the existing literature, we shall try to be internally consistent with our own use of value functions and utility functions as we have defined them."

Keeney & Raiffa\* op.cit.  
Ch. 5.1.1. p. 220 f.n.

As I comprehend their position, the use of utility functions in the context of tradeoffs under rankings of uncertain projects by Keeney and Raiffa can be taken to be synonymous with Von Neumann/Marshak utility functions, which are decidedly cardinal representations of preference orderings.

More precisely, it should be said that such utilities provide a cardinal index of scale measurement. The classic mathematical demonstration of this point (apart from the original works of Von Neumann and Marshak) is in the article by Herstein and Milnor "An Axiomatic Approach to Measurable Utility," Econometrica, 1953.

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\*Also in particular see references Ch. 3.6.2 and Ch. 3.6.6 as well as Ch. 4.1, Ch. 4.2 and Ch. 4.3.

The most complete and thorough treatment of the taxonomy of measuring relative values in a given context of choice, known to me, is that of Peter Fishburn in his Decision and Value Theory, John Wiley & Sons, New York, New York, 1964, of which the following is a partial account.

Consider the problem of assigning relative values of comparison (or rankings) to a set of outcomes (or consequences of an outcome)  $\{O_j\}_{j \in I}$  where  $I$  is a finite index set. One makes the assumption that it is meaningful to ascribe to these outcomes, each, a non-negative real number that represents the relative importance of each outcome to the decision maker. Then for the set  $\{O_j\}_{j \in I}$ , there is a set of numbers in  $\mathbb{R}_+$  such that  $\{O_j\}_{j \in I} \xleftrightarrow{1.1} \{V_j\}$  and  $\forall j \in I \quad V_j \in \mathbb{R}_+$ . Fishburn cites no less than nine alternative means of measuring the  $V_j$ 's, in order of decreasing generality, or in his phraseology, increasing information. I shall restrict myself to merely exhibiting those relationships of ordinal measure and cardinal measure, however, as they are set forth in the tableau that follows.



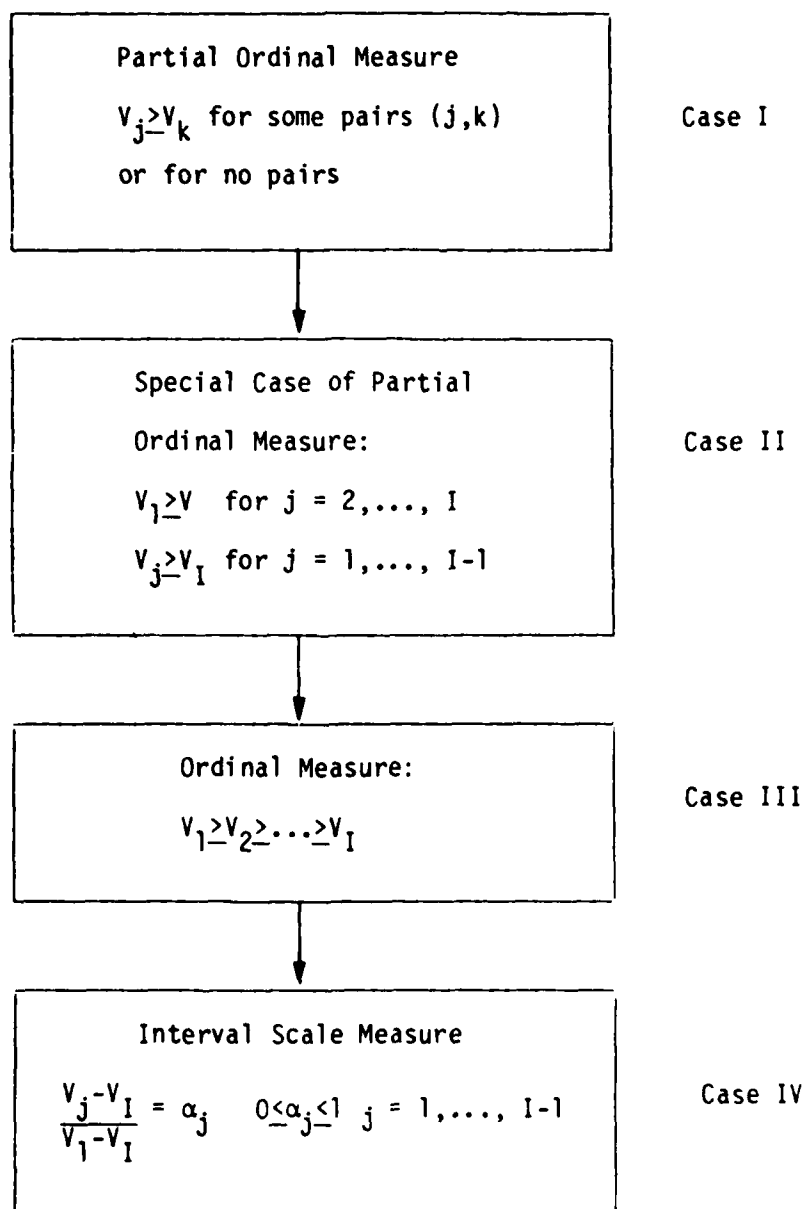


Figure 1

As the diagram shows, Case I  $\rightarrow$  Case II  $\rightarrow$  Case III  $\rightarrow$  Case IV, which has the interpretation that Fishburn gives that the "amount of information" contained in the measure of Case I is also contained in Case II and etc. If we consider the class of value representations of  $\{O_j\}_{j \in I}$ , then in terms of sets, Interval Measure  $\supset$  Ordinal Measure  $\supset$  Special Partial Ordinal Measure  $\supset$  Partial Ordinal Measure. Thus, rather obviously, a refinement of value measure yields more information about the representation. Accordingly, Interval Scale Measures are the most refined of value representations (or preference representations) and thus yield the most unambiguous specification (a comparison of differences) of preferences.

I have three remarks in closing concerning the possible uses of utility analysis to areas of public policy in particular.

The first is that it is widely accepted that some form of interval scale measurement is requisite for the utility framework of the analysis of risk, in order to specify meanings of tradeoffs involving notions of "how much."\*

The second is that when considered in the more general framework of Bernoulli's Principle, forms of the expected utility hypothesis [that is, that an ordering of distributions,  $\{f_{\alpha_j}\}_{\alpha \in A}$ , s.t.  $\sum_j f_j = 1$ , for a given utility representation  $U$ , that ranks outcomes  $\{O_j\} = X$ , can be had by  $W(f_{\alpha}) = \sum_j f_{\alpha_j} U(X_j)$ ] commit one, by implication of certainty equivalents, to assigning utility to amounts of money.\*\*

\*c.f. Guillermo Owen's Game Theory Ch. 4, W. B. Saunders Co., Philadelphia, PA, 1968, pp. 125-126

\*\*c.f. Karl H. Borch's Economics of Uncertainty Ch.'s 1, 2, & 4, Princeton University Press, Princeton, NJ, 1968

The third is that when the decision analysis framework is enlarged to the multi-person context, the conceptual difficulties, and their methodological implications, of making inter-personal comparisons of utility (heterogeneous scale-measures) arise unless some standard of reduction is imposed.\* On this item, game theorists, true to their foundations, usually assume that utility is transferable; which operationally speaking, is equivalent to the existence of a money-like commodity.\*\*

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\*c.f. Luce, Robert D. & H. Raiffa's Games and Decisions: Introduction and Critical Survey Ch. 2.7, Wiley & Sons, New York, NY, 1957

\*\*c.f. Shapley, L. S. & M. Shubik "Quasi-Cores of Monetary Economists with Nonconvex Preferences," Econometrica, 1966

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